Generalized Theory for Nanoscale Voltammetric Measurements of Heterogeneous Electron-Transfer Kinetics at Macroscopic Substrates by Scanning Electrochemical Microscopy

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Supporting Information

ABSTRACT: Here we report on a generalized theory for scanning electrochemical microscopy to enable the voltammetric investigation of a heterogeneous electron-transfer (ET) reaction with arbitrary reversibility and mechanism at the macroscopic substrate. In this theory, we consider comprehensive nanoscale experimental conditions where a tip is positioned at a nanometer distance from a substrate to detect the reactant or product of a substrate reaction at any potential in the feedback or substrate generation/tip collection mode, respectively. Finite element simulation with the Marcus—Hush—Chidsey formalism predicts that a substrate reaction under the nanoscale mass transport conditions can deviate from classical Butler—Volmer behavior to enable the precise determination of the standard ET rate constant and reorganization energy for a redox couple from the resulting tip current—substrate potential voltammogram as obtained at quasi-steady state. Simulated voltammograms are generalized in the form of analytical equations to allow for reliable kinetic analysis without the prior knowledge of the rate law. Our theory also predicts that a limiting tip current can be controlled kinetically to be smaller than the diffusion-limited current when a relatively inert electrode material is investigated under the nanoscale voltammetric conditions.

Scanning electrochemical microscopy (SECM) has been successfully applied for the investigation of heterogeneous electron-transfer (ET) reactions at various electroactive substrates comprised of metals, semiconductors, and carbons as well as those modified with monolayers and polymers.1,2 Most of these SECM studies of important electrode materials are based on steady-state feedback measurements.3 Steady-state conditions facilitate imaging of heterogeneous substrate surfaces and kinetic mapping of “hot” spots and simplify the measurement and analysis of an approach curve, a plot of tip current versus tip—substrate distance, from which a local ET rate constant can be determined conveniently using analytical equations.4 The feedback mode is also useful for the quantitative study of heterogeneous ET kinetics at unconventional substrates such as unbiased conductors,5 air/water interfaces,6 liquid/liquid interfaces,7 and biological cells.8 SECM has also emerged as a vital tool in nanoelectrochemistry9,10 to realize unprecedentedly fast kinetic measurements of electrode reactions under extremely high mass transport conditions. The powerfulness of the feedback mode with nanometer distances between the SECM tip and the substrate was demonstrated in the kinetic study of rapid ET reactions at nanotips,11,12 the electrogeneration and detection of short-lived intermediates,13 and the electrochemical detection of single molecules.14—16 Alternatively, nanoscale mass transport conditions can be achieved even with micrometer tip—substrate distances when individual nanobands17—19 and single-walled carbon nanotubes20 are investigated as SECM substrates.

Recently, we developed a powerful SECM method based on both feedback and substrate generation/tip collection (SG/TC) modes to enable quasi-steady-state voltammetry of rapid ET reactions at macroscopic substrates by employing nanometer tip—substrate distances.21 With this method, the reactant, O, or product, R, of a reversible substrate reaction (O + e ⇌ R) is amperometrically detected at the tip in the feedback or SG/TC mode, respectively, during the cyclic sweep of substrate potential, E0, across the formal potential of the redox couple, Eredox. Despite the voltammetric growth of a planar diffusion layer at the macroscopic substrate, the tip current, iT, in either operation mode immediately reaches quasi-steady state at nanometer tip—substrate distances, where concentrations of a reversible redox couple are always equilibrated with substrate potential. Moreover, the resulting high mass transport of redox mediators across the narrow tip—substrate gap kinetically limits the rapid ET reaction at the substrate surface under the tip. Advantageously, a pair of quasi-steady-state iT—E0 voltammograms thus obtained in feedback and SG/TC modes allow for the reliable determination of kinetic and thermodynamic parameters in the Butler—Volmer (BV) model as well as all transport parameters. A standard ET rate constant, k0, of 7 cm/s was determined by nanoscale iT—E0 voltammetry at tip—substrate distances of ~90 nm as one of the largest k0 values reported for heterogeneous ET reactions.10,22

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contrast, we predict in this work that, except for these fastest ET reactions, the BV model is not appropriate under high mass transport conditions as practiced for the various nanoscale SECM measurements.

Here we report on a generalized theory for nanoscale \( i_{\text{r}}-E_{\text{s}} \) voltammetry of substrate reactions with arbitrary reversibility and mechanism under comprehensive experimental conditions including any substrate potential and both feedback and SG/TC modes. We employ the Marcus–Hush–Chidsey (MHC) formalism for heterogeneous outer-sphere ET reactions\(^{23-25} \) to enable time-dependent finite element simulations of nanoscale \( i_{\text{r}}-E_{\text{s}} \) voltammograms for various kinetic regimes beyond the limit of BV kinetics. The MHC model is more realistic at large overpotentials where ET rate constants become smaller than predicted by BV kinetics and eventually become potential-independent in contrast to classical Marcus “inverted” behavior.\(^{26} \) This limiting behavior at extreme overpotentials is due to the continuum of electronic states in the electrode and was observed by chronoamperometry of redox centers attached to metal and carbon ultramicroelectrodes.\(^{27} \) Also, the manifestation of a limiting ET rate constant for a diffusional system has been theoretically predicted for steady-state voltammetry at disk nanaoelectrodes\(^{24} \) and single-walled carbon nanotubes\(^{28} \) and also reported for cyclic voltammetry of outer-sphere redox couples at gold electrodes modified with self-assembled monolayers of insulating alkanethiols.\(^{29} \)

Importantly, nanoscale \( i_{\text{r}}-E_{\text{s}} \) voltammograms thus simulated for both feedback and SG/TC modes are generalizable in the form of analytical equations. These equations allow for the determination of an ET rate constant from a tip current without the prior knowledge of the rate law to reveal the potential-dependence of both cathodic and anodic processes at the substrate from the paired voltammograms. The feasibility of the SECM observation of MHC voltammetric responses at relatively inert electrode materials is theoretically assessed using the numerical and analytical approaches with kinetic parameters as determined by cyclic voltammetry of outer-sphere redox couples at the low-defect basal plane surface of highly oriented pyrolytic graphite (HOPG).\(^{30} \)

**MODEL**

As reported elsewhere,\(^{21} \) our model is defined in a cylindrical coordinate, where a disk-shaped SECM tip is faced in parallel to the surface of a disk-shaped substrate. The disk radius of the macroscopic substrate is 50 times larger than the tip radius, \( a \), and is large enough not to affect simulation results. Feedback and SG/TC modes of SECM are employed to monitor a first-order one-electron process at a macroscopic substrate as given by

\[
O + e \xrightarrow{k_{\text{f}}} \text{R}
\]

where \( k_{\text{f}} \) and \( k_{\text{b}} \) are first-order heterogeneous ET rate constants. The cathodic rate constant is defined by the MHC formalism as\(^{24} \)

\[
k_{\text{f}} = k^0 \exp \left( \frac{E^*}{2} \right) \int_{-\infty}^{\infty} \frac{\exp \left[ -\frac{(\epsilon^* - E^*)^2}{4 \lambda^2} \right]}{2 \cosh \left[ \frac{\epsilon^*}{2} \right]} \, d\epsilon^*
\]

\[
\lambda^* = \frac{\lambda}{k_{\text{b}} T}
\]

\[
\epsilon^* = \frac{\epsilon}{k_{\text{b}} T}
\]

where \( \lambda \) (electronvolts) is the reorganization energy of the redox couple and \( \epsilon \) is an integration variable. The potential-dependence of \( k^0 \) in this model implies that the density of states in the electrode is constant and independent of the potential and that the electronic interaction between a redox molecule and each energy level in the electrode is independent of the energy level and of the neighboring levels.\(^{31} \) Alternatively, the BV model gives

\[
k_{\text{b},f} = \frac{k^0}{\lambda^*} \exp(-\alpha \epsilon^*)
\]

where \( \alpha \) is the transfer coefficient. In either model, the Nernst equation must be satisfied at equilibrium, which requires\(^{25} \)

\[
k_{\text{b}} = k_{\text{f}} \exp(\epsilon^*)
\]

Finite element simulations were carried out using COMSOL Multiphysics (version 4.1, COMSOL, Inc., Burlington, MA) linked to Matlab (version 2010b, MathWorks, Natick, MA). The potential-dependence of cathodic (eq 2) and anodic (eq 7) rate constants, i.e., \( k_{\text{f},\text{b}}/k^0 \) and \( k_{\text{b},\text{f}}/k^0 \), respectively, were evaluated for various \( \lambda^* \) values using Matlab and Mathematica (version 7, Wolfram Research, Inc., Champaign, IL) to find that outcomes of these two approaches agree very well with each other (Figure 1). Moreover, calculated \( k_{\text{lim}}/k^0 \) and \( k_{\text{b},\text{f}}/k^0 \) values approach to limiting values, \( k_{\text{lim}}/k^0 \), reported by Feldberg as\(^{24} \)

\[
k_{\text{lim}} = \frac{1}{4 \pi \lambda^*} \exp \left( \frac{\lambda^*}{4} \right)
\]

Our results also confirm that >99% of \( k_{\text{lim}} \) is achieved when overpotential (equivalent to \( E_s - E^* \) in this work) is large enough with respect to \( \lambda^* \) to satisfy the following condition\(^{24} \)

\[
|E_s - E^*| \geq 1.35\lambda^* + \frac{7.7RT}{F}
\]

The potential-dependent parts of rate constants thus calculated using Matlab were called externally from COMSOL Multiphysics through LiveLink Matlab to define the boundary condition at the substrate surface. Two-dimensional, time-dependent diffusion problems for SECM were solved using the following dimensionless parameters as reported elsewhere\(^{21} \)

\[
\lambda_{O} = \frac{k^0 d}{D_O} \quad \text{dimensionless standard ET rate constant}
\]

\[
L = \frac{d}{a} \quad \text{dimensionless tip–substrate distance}
\]
\[ \sigma = a^2 F v/4D_0RT \]  
\text{(dimensionless sweep rate for substrate potential)} \tag{12} \\
\[ \xi = \sqrt{D_O/D_R} \]  
\text{(dimensionless diffusion coefficient ratio)} \tag{13} 

where \( d \) is the tip–substrate distance, \( v \) is the sweep rate for substrate potential, and \( D_O \) and \( D_R \) are diffusion coefficients of species O and R, respectively, in the bulk solution. Unfortunately, the “simulation report” cannot be generated using version 4.1 of COMSOL Multiphysics. Simulation files used for this work are available upon request.

A tip current was calculated for the diffusion-limited amperometric detection of the original mediator, O, in the feedback mode or the substrate-generated species, R, in the SG/TC mode to give a pair of \( i_T-E_S \) voltammograms at the same tip–substrate distance. A tip current was obtained in the normalized form (eq S-1 in the Supporting Information) with respect to the diffusion-limited current for species O at the tip in the bulk solution, \( i_T(\infty) \), as given by

\[ i_T(\infty) = 4\pi n F D_O c_O \frac{a}{a} \]  
\text{(14)}

where \( RG \) is expressed by eq S-2 in the Supporting Information as a function of RG (= \( r_e/a; r_e \) is the outer tip radius, \( RG = 1.5 \), and correspondingly, \( x = 1.16 \) are employed in this work), \( n = 1 \) for a one-electron process considered in this model (eq 1), and \( c_O^* \) is the concentration of the original mediator in the bulk solution. The \( x \) values simulated for different RG values at \( L = 50 \) agree with theoretical values given by eq S-3 in the Supporting Information, thereby validating our simulation results. The dimensionless flux of species O at the macroscopic substrate represents a substrate current. Positive tip (and substrate) currents are based on reduction of the original mediator. Subsequently, feedback and SG/TC tip responses appear in the upper and lower panels of the following graphs, respectively.

### RESULTS AND DISCUSSION

**Simulations of Nanoscale \( i_T-E_S \) Voltammograms Based on the MHC Formalism.** Dimensionless kinetic parameters, \( \lambda^* \) (eq 4) and \( \lambda_{O^*}^* \) (eq 10), in the MHC formalism were systematically varied to simulate \( i_T-E_S \) voltammograms for various kinetic regimes at nanoscale tip–substrate distances (Figure 2 and Figures S-1 and S-2 in the Supporting Information for \( \lambda^* = 40, 60, \) and 20, respectively). These \( \lambda^* \) values correspond to realistic \( \lambda \) values of 0.5–1.5 eV at 298 K (i.e., 12–36 kcal/mol) as estimated theoretically or determined experimentally for various outer-sphere redox mediators (Table S-1 in the Supporting Information). Characteristic features of the MHC behavior were fully demonstrated by varying \( \lambda_{O^*}^* \) in a wide range, e.g., from 10 to 10^{-8} for \( \lambda^* = 40 \) (Figure 2). In addition, the tip–substrate distance and time scale of the simulations were set to \( L = 0.25 \) and \( a = 4 \times 10^{-4} \), respectively, which are small enough to give quasi-steady-state \( i_T-E_S \) voltammograms at any \( E_S \) when the reaction at the macroscopic substrate is reversible. These small \( L \) and \( a \) values can be achieved experimentally using an in situ ink deposition with a submicrometer or nanometer tip radius. For instance, a sharp SECM tip with \( a \approx 0.5 \) \( \mu \)m and \( RG = 1.5 \)–2.5 can be readily positioned within \( L = 0.25 \) (\( d = 0.125 \) \( \mu \)m in eq 11) from the substrate surface. This small \( L \) value is also achievable using carefully polished and aligned nanotips. With \( a \approx 0.5 \) \( \mu \)m, the cyclic sweep of substrate potential at \( v \leq 100 \) mV/s is equivalent to \( \sigma \leq 4 \times 10^{-4} \) for a redox mediator with a typical \( D_O \) value of \( 5 \times 10^{-6} \) cm/s (see eq 12). In addition, \( D_O = D_R (\xi = 1) \) was assumed for simplicity although a reversible redox couple with \( D_O \neq D_R \) gives a quasi-steady-state \( i_T-E_S \) voltammogram in either feedback or SG/TC mode with such small \( \sigma \) and \( RG (= 1.5) \).

Simulation results with either BV\textsuperscript{11} or MHC model demonstrate that the reversibility of the reaction at the macroscopic substrate controls substrate potentials where a steady-state or quasi-steady-state tip current is obtained in \( i_T-E_S \) voltammetry. The whole \( i_T-E_S \) voltammogram in either feedback or SG/TC mode is obtained under steady-state or quasi-steady-state conditions to be sigmoidal and completely retraceable at any substrate potential on its cyclic sweep (\( \lambda_{O^*}^* = 10 \) and 1 in Figure 2) when the reaction at the macroscopic substrate is
reversible as confirmed by simulated cyclic voltammograms (CVs) at the substrate (Figure S-3 in the Supporting Information). Note that the substrate reaction with $\lambda_{O}' = 1$ is locally quasi-reversible under the tip, thereby yielding a broader $i_{R} - E_{S}$ voltammogram than the reversible counterpart with $\lambda_{O} = 10$. On the other hand, the tip current does not always reach to steady state or quasi-steady state when the reaction at the macroscopic substrate is kinetically limited to give a non-Nernstian substrate CV ($\lambda_{O}' \leq 0.1$ in Figure S-3 in the Supporting Information). The corresponding $i_{R} - E_{S}$ voltammograms (Figure 2) are retraceable only if substrate potentials are far enough from $E^{0}$ to drive the reaction at the macroscopic substrate to a diffusion limit. Irretraceable portions of the voltammograms around $E_{S} = E^{0}$ are transient and give higher current on the forward (or reverse) potential sweep in the feedback (or SG/TC) mode. The substantial current in the feedback mode around $E_{S} = E^{0}$ on the forward sweep is due to negative feedback effect at the tip, where mediator molecules diffuse from the bulk solution to be electrolyzed without being regenerated at the practically inert substrate surface in this potential range. In the SG/TC mode, mediator molecules are generated at the whole substrate surface on the forward sweep to be detected at the tip on the reverse sweep, where the magnitude of the tip current around $E_{S} = E^{0}$ is close to that of the negative feedback current expected at this tip–substrate distance.

Unique voltammetric features of MHC responses are seen in retraceable portions of $i_{R} - E_{S}$ voltammograms. These features are similar to those predicted for steady-state voltammograms at disk electrodes with the exception that they are completely sigmoidal without an irreversible portion. Specifically, the retraceable portion of an $i_{R} - E_{S}$ voltammogram based on the MHC mechanism becomes broader than that based on the BV mechanism with $\alpha = 0.5$ when a substrate reaction is slow enough, for instance, $\lambda_{O}' < 10^{-1}$ with $\lambda^{*} = 40$ (Figure 2). Importantly, this requirement is achieved even with relatively fast substrate reactions under nanoscale mass transport conditions, e.g., $\lambda_{O}' < 10^{-1}$ in eq 9 with $D_{O} = 5 \times 10^{-6}$ cm$^2$/s corresponds to $k_{D} < 0.1$ and $< 1$ cm/s at $d = 50$ and 5 nm, respectively. In other words, the BV model breaks down at $d = 5$ nm except for the fastest ET reactions ($k_{D} > 1$ cm/s) reported so far.

More strikingly, the limiting current of an $i_{R} - E_{S}$ voltammogram for a slower reaction, for instance, $\lambda_{O}' < 10^{-3}$ with $\lambda^{*} = 40$, becomes smaller than the diffusion-limited current as observed with $\lambda_{O}' < 10^{-3}$. This smaller limiting current is controlled by $k_{lim}$ at extreme substrate potentials (see eqs 8 and 9). Interestingly, this result indicates that an approach curve to a metal at this kinetic limit is more negative than the diffusion-limited positive feedback curve (see below). Overall, broad shapes and low kinetic limiting currents of steady-state and quasi-steady-state MHC responses are observed more readily for a redox couple with smaller $\lambda^{*}$, subsequently, with larger $\lambda_{O}'$ and at less extreme potentials (see Figures S-1 and S-2 in the Supporting Information for $\lambda^{*} = 60$ and 20, respectively).

Noticeably, retraceable portions of $i_{R} - E_{S}$ voltammograms in feedback and SG/TC modes display inversion symmetry against $E^{0}$ (Figure 2 and Figures S-1 and S-2 in the Supporting Information) as expected from the reciprocity relation of $k_{S}(E^{0}) = k_{S,b}(-E^{0})$ in the MHC formalism (see eqs 2 and 7). In contrast, retraceable portions of $i_{R} - E_{S}$ voltammograms based on the BV mechanism display inversion symmetry only if $\alpha = 0.5$ (Figure S-4 in the Supporting Information). In either mechanism, inversion symmetry is compromised in irretraceable portions around $E_{S} = E^{0}$ due to concentration inequalities between the feedback and SG/TC modes.

**Determination of Kinetic Parameters from MHC Responses.** Here we demonstrate that $\lambda_{O}' = \lambda^{*}$ (and subsequently $k_{D}$ and $\lambda^{*}$) in the MHC mechanism can be determined precisely by nanoscale $i_{R} - E_{S}$ voltammetry when a reaction at the macroscopic substrate is slow enough to reveal the dependence of its rate on reorganization energy at large overpotentials without a diffusion limit. The specific conditions required for the complete kinetic analysis of a MHC response is assessed by comparing $i_{R} - E_{S}$ voltammograms of redox couples with $\lambda^{*} = 40$ and 60 in the feedback mode. Such a comparison of the corresponding voltammograms in the SG/TC mode does not provide additional kinetic information because of the inversion symmetry of the paired voltammograms.

The precision of $\lambda_{O}'$ and $\lambda^{*}$ values that are determined from a MHC voltammetric response depends on the range of substrate potentials where the retraceable portion of an $i_{R} - E_{S}$ voltammogram is observed. With $\lambda_{O}' = 0.1$ (Figure 3a), $i_{R} - E_{S}$ voltammograms with $\lambda^{*} = 40$ and 60 (circles and solid line, respectively) are indistinguishable and barely broader than the $i_{R} - E_{S}$ voltammogram based on BV kinetics with $\alpha = 0.5$ (dotted line). In this kinetic regime, only $\lambda_{O}'$ can be determined uniquely from an $i_{R} - E_{S}$ voltammogram, which is independent of $\lambda^{*}$. This result confirms that the MHC mechanism and the BV mechanism with $\alpha = 0.5$ are equivalent in the potential region in the vicinity of $E^{0}$ (see Figure 1). As $\lambda_{O}'$ decreases to $10^{-2}$ and then to $10^{-3}$ (parts b and c of Figure 3, respectively), an $i_{R} - E_{S}$ voltammogram based on the MHC mechanism becomes broader and shifts toward more extreme potentials than expected from the BV mechanism with $\alpha = 0.5$. Moreover, an $i_{R} - E_{S}$ voltammogram with smaller $\lambda^{*}$ is broader for a given $\lambda_{O}'$. Nevertheless, $i_{R} - E_{S}$ voltammograms with $\lambda^{*} = 40$ and 60 are nearly indistinguishable for $\lambda_{O}' = 10^{-2}$ because they overlap with each other when slightly smaller $\lambda_{O}'$ is employed for $\lambda^{*} = 60$ (dashed line). In contrast, such adjustment does not superimpose $i_{R} - E_{S}$ voltammograms with $\lambda^{*} = 40$ and 60 for a slower substrate reaction with $\lambda_{O}' = 10^{-3}$, thereby enabling the separate determination of $\lambda_{O}'$ and $\lambda^{*}$. Eventually, $\lambda^{*}$ can be determined most precisely when $\lambda_{O}'$ is small enough to kinetically control...
A limiting current. With $\lambda^{*} = 10^{-4}$ (Figure 3d), an $i_T - E_S$ voltammogram with $\lambda^{*} = 40$ gives a kinetic limiting current at substrate potentials where the diffusion-limited current is observed for $\lambda^{*} = 60$ in the range of $\lambda_{0}^{*} = 10^{-4} - 10^{-5}$. Moreover, the $i_T - E_S$ voltammograms with $\lambda^{*} = 60$ are much steeper than the voltammogram with $\lambda^{*} = 40$.

Overall, the MHC response that is unique with a typical $\lambda^{*}$ value of 40 is seen with $\lambda_{0}^{*} \leq 10^{-3}$, which corresponds to $k^0 \leq 10^{-3}$ cm/s in eq 10 with $D_O = 5 \times 10^{-6}$ cm$^2$/s and $d = 50$ nm. Such low $k^0$ values have been reported for outer-sphere redox mediators at relatively inert electrode materials such as HOPG (Table S-1 in the Supporting Information). In addition, reliable kinetic analysis requires the potential window that is wide enough to observe at least the retraceable portion of a broad voltammogram based on the MHC mechanism. These requirements are assessed below.

The accurate determination of kinetic parameters from an $i_T - E_S$ voltammogram based on the MHC mechanism requires the discrimination of a MHC response from a BV response. In fact, a steady-state voltammogram based on the MHC mechanism can be fitted with that based on the BV mechanism to yield erroneous $\alpha$ and $k^0$ values.\(^{24}\) Importantly, a MHC response can be unambiguously discriminated from a BV response by measuring a pair of $i_T - E_S$ voltammograms in both feedback and SG/TC modes. Such a pair of voltammograms based on the MHC mechanism display inversion symmetry against $E^0$ in contrast to the corresponding pair of $i_T - E_S$ voltammograms based on the BV mechanism with $\alpha \neq 0.5$. Specifically, a broad $i_T - E_S$ voltammogram based on the MHC mechanism in the feedback (or SG/TC) mode can be fitted well using the BV model with incorrectly large $k^0$ and anomalously large (or small) $\alpha$, which compromises inversion symmetry (Figure 4a). Subsequently, the corresponding MHC voltammogram in the SG/TC (or feedback) mode cannot be fitted with the much steeper voltammogram based on the BV model, thereby revealing its inappropriateness in this kinetic regime. Advantageously, a pair of $i_T - E_S$ voltammograms in feedback and SG/TC modes are obtained simply by setting the tip potential within an appropriate range in the presence of only the oxidized or reduced form of a redox couple in the bulk solution.\(^{21}\) In contrast, both oxidized and reduced forms of a redox couple must be prepared and stable in the bulk solution to obtain a pair of steady-state voltammograms at ultramicroelectrodes, which was proposed to avoid the erroneous analysis of a MHC response using the BV model.\(^{24}\) Noticeably, reliable kinetic analysis using a pair of steady-state voltammograms was demonstrated both theoretically\(^{36}\) and experimentally\(^{37,38}\) for nanopipet voltammetry of rapid ion transfers at liquid/liquid interfaces.

Additionally, the irretraceable portion of an $i_T - E_S$ voltammogram is useful for the discrimination of a MHC response from a BV response when a substrate reaction is slow enough to give a plateau current around $E_S = E^0$ (Figure 4b). Since a slower substrate reaction gives the low plateau current in a wider range of substrate potentials, irretraceable portions of $i_T - E_S$ voltammograms based on MHC and BV mechanisms are not superimposed when erroneously large $\lambda_{0}^{*}$ is employed in the BV model to fit retraceable portions with each other (Figure 4b). Moreover, this fitting procedure requires the adjustment of the diffusion-limited current of the BV response to the kinetic limiting current of the MHC response, which is detected as an error in the tip—substrate distance.\(^{21}\) Overall, the discrimination of a MHC response from a BV response in this slow kinetic regime requires only the feedback or SG/TC mode, which is useful when only one of the very broad voltammograms is observable within the potential window.

It should be noted that the symmetric broadening of a pair of $i_T - E_S$ voltammograms due to the MHC effect can be discriminated from their asymmetric broadening due to the effect of an electrical double layer on a substrate reaction. The asymmetric double-layer effect on rates of forward and reverse substrate reactions results from (i) different charges of oxidized and reduced forms of a redox couple and (ii) different potentials for their electrolysis at the outer Helmholtz plane.\(^{39}\) For instance, the double-layer effect is negligible in the feedback (or SG/TC) mode based on the substrate reaction of an electrically neutral form but can be significant in the SG/TC (or feedback) mode based on the reverse reaction of the other form, which must be charged. A distinction between MHC and double layer effects illustrates the advantage of a pair of steady-state (or quasi-steady-state) voltammograms based on forward and reverse reactions against a single sigmoidal voltammogram dominated by one of the reactions.

**Analytical Equations for Nanoscale $i_T - E_S$ Voltammograms.** The retraceable portion of an $i_T - E_S$ voltammogram in either the feedback or SG/TC mode is obtained under steady-state or quasi-steady-state conditions and, subsequently, can be represented by analytical equations to facilitate its kinetic analysis even when the rate law is unknown (see below). In the steady-state feedback mode, the normalized tip current, $I_T(L,E_S)$, is given by\(^{4}\)

$$I_T(L,E_S) = \frac{I_{T_{PF}}(L + 1/A, RG)}{(1 + 2.47RG^{0.31}LA)(1 + L^{-0.0068G + 0.1133A - 0.0236RG + 0.91})}$$  \hspace{1cm} (15)

with

$$A = \frac{k_{s,LA}}{D_O}$$  \hspace{1cm} (16)
where $I_{T,PF}$ and $I_{T,NE}$ represent normalized tip currents in purely positive and negative potential conditions and are given by eqs S-3 and S-5 in the Supporting Information, respectively. Equation 15 fits very well with retraceable portions of the voltammograms simulated for the feedback mode in slow kinetic regimes including those with kinetic limiting currents (top panels of Figure S-5 in the Supporting Information). For this comparison, potential-dependent $k_{SB}$ values in eq 15 were calculated from a combination of eqs 2 and 7 using Mathematica (see Figure 1). Interestingly, eq 15 is also applicable for the SG/TC mode when $k_{SB}$ is replaced by $k_{SF}$ in eq 16 and $D_O = D_R$ ($\xi = 1$) is assumed (bottom panels of Figure S-5 in the Supporting Information). Overall, good fits as obtained with eq 15 demonstrate that an $I_T = E_S$ voltammogram in either operation mode is retraceable when a substrate reaction under the tip is totally irreversible and opposite to the tip reaction. In addition, eq 15 agrees well with low plateau currents around 0.25 as used for their derivations.

We also propose the following analytical equations for feedback and SG/TC modes, which are more useful than eq 15 when an ET mechanism is unknown (see the Supporting Information for their derivations)

$$I_{T,PF}^{FB}(E_S, L) = \frac{\pi}{2xL(2/\theta_S + 2D_O/k_{SB}d + \xi^2 + 1)}$$

$$+ \frac{\xi\theta_S}{\xi\theta_S + 1} \left[ I_{T,lim}(L) - \frac{\pi}{2xL(2D_O/k_{lim}d + \xi^2 + 1)} \right]$$

$$I_{T,NE}^{ST}(E_S, L) = - \left\{ \frac{\pi}{2xL(2\xi^2/\theta_S + 2D_O/k_{SB}d + \xi^2 + 1)} \right\}$$

$$+ \frac{1}{\xi\theta_S + 1} \left[ \frac{\pi}{2xL(2D_O/k_{lim}d + \xi^2 + 1)} \right]$$

$$I_{T,PF}^{FB}(E_S, L)$$ and $I_{T,NE}^{ST}(E_S, L)$ represent normalized tip currents for the respective operation modes, $\theta_S = \exp(E^*)$, and $I_{T,lim}(L)$ is the normalized limiting current. In comparison to eq 15, eqs 17 and 18 can be more easily solved for $k_{SB}$ and $k_{SF}$ as functions of the tip current while $k_{lim}$ must be determined from $I_{T,lim}(L)$ using eq 15. Subsequently, a pair of plots of $k_{SB}$ or $k_{SF}$ versus $E_S$, e.g., Figure 1, can be obtained from a pair of $I_T = E_S$ voltammograms in the feedback and SG/TC modes to reveal the potential-dependence of both anodic and cathodic processes at the substrate. Noticeably, eqs 17 and 18 are valid under steady-state and quasi-steady-state conditions and do not reproduce retraceable portions of a voltammogram (Figure S-5 in the Supporting Information).

Analytical equations are useful for the analysis of approach curves, for which eq 15 was originally applied.

Figure 5 shows the approach curves generated using eq 15 with the MHC formalism, where $k_{SB}$ values at various overpotentials were calculated using eqs 2 and 7 with $\lambda^* = 40$. In this example, the slow substrate reaction that corresponds to $\lambda_O = 10^{-3}$ at $d/a = 0.25$ as used for curves $g$ and $g'$ in Figure 1 was considered to emphasize the difference between MHC and BV mechanisms. As overpotential becomes more positive, i.e., $A$ becomes larger, the corresponding approach curve based on the MHC mechanism becomes more positive and then reaches to a kinetically limited curve (curve a), which is more negative than the purely positive feedback curve expected for the BV mechanism (dotted line).

**Experimental Observation of MHC Limiting Current.** Here we consider the low-defect basal plane surface of HOPG as a model substrate to quantitatively predict that the kinetic limiting current of a MHC voltammetric response will be observable at such a relatively inert electrode by employing nanoscale $I_T - E_S$ voltammetry. Estimated $\lambda$ values and experimental $k'$ values are obtained for various redox mediators at the HOPG surface were used to obtain $k_{lim}$ from eq 8 by assuming that the MHC formalism is valid also for semimetallic HOPG. With a $k_{lim}$ value, normalized limiting currents, $I_{T,lim}$ at $L = 0.25$ were calculated using eq 15 with $a = 20$ and 200 nm, i.e., $d = 5$ and 50 nm, respectively, and $D_O = 5 \times 10^{-6}$ cm$^2$/s. The comparison of $I_{T,lim}$ with the diffusion-limited $I_{T,PF}$ value (Table S-1 in the Supporting Information) indicates that mass transport across 5–50 nm-wide tip–substrate gaps is efficient enough to exert significant kinetic effect on a limiting current for redox mediators with $\lambda = 0.6–1.3$ eV including MV$^{2+/+}$ (MV = methylviologen), IrCl$_6^{3-/2-}$, Fe(COOH)$_2$ (1,1'-dicarboxyferrocene), W(CN)$_6^{3-/4-}$, Ru(CN)$_6^{3-/4-}$, and Fe(CN)$_6^{3-/4-}$. Moreover, broad $I_T - E_S$ voltammograms of these mediators in the feedback (or SG/TC) mode are observable within the potential limit of HOPG, which is wider than +1 V versus SCE and satisfies eq 9 for these mediators in the corresponding operation mode (Table S-1 in the Supporting Information). In contrast, Ru(NH$_3$)$_6^{3+/2+}$ and Co(phen)$_3^{3+/2+}$ (phen = 1,10-phenanthroline) with larger $\lambda$ values of 1.4 and 1.6 eV, respectively, require a narrower gap of <5 nm, where the tip current may be significantly biased by electron tunneling between the tip and the substrate. Also, Ru(bpy)$_3^{3+/2+}$ (bpy = 2,2'-bipyridyl) and Fe(phen)$_3^{3+/2+}$ with small $\lambda$ values of 0.5 and 0.6 eV, respectively, need short tip–substrate distances of <5 nm because of their high intrinsic reactivity, which varies with defect density at the HOPG surface.
slow outer-sphere ET reactions at HOPG or other macroscopic electrodes. To demonstrate this limitation, simulated substrate CVs based on the MHC mechanism were fitted with broad experimental CVs of the Fe(CN)\(6^{3-/-4-}\) couple at macroscopic HOPG electrodes reported in the seminal work by McCready and co-workers. For instance, the best fit for a CV at 10 V/s was obtained using \(\lambda^*=32\) (\(\lambda = 0.82\) eV at 298 K) and \(k_0^*=5.6 \times 10^{-6}\) cm/s while the totally irreversible CV based on BV kinetics with \(\lambda = 0.5\) is much narrower (Figure 6). The fit as obtained using the MHC formalism is as good as demonstrated in McCready’s work by assuming the linear potential-dependence of \(\lambda\) in the BV model. In fact, the MHC formalism gives a linear relationship between \(E_S\) and the apparent transfer coefficient, \(\alpha_{app}\) (see inset of Figure 6), when \(\alpha_{app}\) is defined as:

\[
\alpha_{app} = \frac{\text{RT ln}(k_{S\lambda}/k_0)}{F(E_S - E_D^\lambda)} = 1 - \frac{\text{RT ln}(k_{S\lambda}/k_0)}{F(E_S - E_D^\lambda)}
\]

In eq 18 with the MHC formalism, \(\lambda^*=32\) corresponds to \(\text{d}\alpha_{app}/\text{d}E_S = 0.25\) V\(^{-1}\) as found in the original work from the analysis of the experimental CV in Figure 5. This linear relationship, however, is compromised as \(E_S\) approaches the region where \(k_{lim}\) is achieved. Therefore, the linear potential-dependence of \(\alpha_{app}\) which has been also reported for various electrode reactions and interpreted by the classical Marcus theory with an inverted region, indicates that \(k_{lim}\) has not been reached without a diffusion limit under moderate mass transport conditions at macroscopic substrates. In fact, the range of substrate potentials swept in Figure 6 is much narrower than required for achieving \(k_{lim}\) for \(\lambda^*=32\), which corresponds to \([E_S - E_D^\lambda] \geq 1.3\) V in eq 9 at 298 K. These results support that higher mass transport conditions in nanoscale \(i_T-E_S\) voltammetry are required for the experimental observation of a kinetic limiting current. Alternatively, nanoscale mass transport is achieved at the nanometer-sized basal plane surface of HOPG, which is difficult to prepare without the serious formation of defects.

In addition to the MHC effect, the diffusional effect is likely to play a role in broadening of CVs at HOPG. Experimental CVs of the Fe(CN)\(6^{3-/-4-}\) couple are significantly broader than CVs simulated with a \(\lambda^*\) value of 51 as theoretically estimated for HOPG although smaller \(\lambda^*\) values of 27-44 (\(\lambda = 0.7-1.13\) eV) were experimentally determined for other systems (Table S-1 in the Supporting Information). This result suggests that the experimental CVs are broadened due to the diffusional effect within various domains at the heterogeneous surface of HOPG as quantitatively assessed using the diffusion domain approach. In fact, the limitation of HOPG in the reproducible experimental test of the MHC formalism is the large variation of grain size and structure among samples even from the same source as confirmed by focused-ion-beam imaging of their surfaces.

### CONCLUSIONS

Here we employed the MHC formalism to theoretically predict that MHC voltammetric behavior will be manifested by SECM-based \(i_T-E_S\) voltammetry under nanoscale mass transport conditions. Our voltammetric theory is generally applicable to any steady-state and quasi-steady-state condition with arbitrary substrate potential and both the feedback and SG/TC modes. Therefore, this deviation from the classical BV reactions reported so far (\(k^0 > 1\) cm/s). Our prediction is also extended to steady-state voltammetry of ET reactions at nanometer-sized SECM tips positioned above conductors because tip voltammetry can be well described by the general theory that is analogous to the theory for \(i_T-E_S\) voltammetry. In comparison to nanotips, macroscopic substrates can be fabricated from a much larger variety of electroactive materials to be investigated by SECM, thereby augmenting the significance of our theory.

Steady-state and quasi-steady-state MHC responses in \(i_T-E_S\) voltammetry are recognized not only as a broad wave shape but also as a kinetically limited plateau current, which contrasts to classical Marcus “inverted” behavior. The experimental observation of a limiting ET rate constant without a diffusion limit will be possible under high mass transport conditions as achieved at nanoscale tip–substrate gaps in SECM. On the other hand, our theory indicates that the broad wave shape of a MHC response corresponds to the linear potential-dependence of the apparent transfer coefficient in the BV model as observed with various macroscopic electrodes and interpreted by the classical Marcus theory. Importantly, this linear potential-dependence is the evidence that a limiting ET rate constant due to the MHC mechanism is not reached. In addition, the erroneous analysis of a broad \(i_T-E_S\) voltammogram using the BV model can be avoided by measuring a pair of voltammograms in feedback and SG/TC modes, which display inversion symmetry as another important feature of MHC voltammetric behavior under steady-state and quasi-steady-state conditions.

No prior knowledge of the rate law is required for the analysis of the retraceable portion of an \(i_T-E_S\) voltammogram, which can be directly converted to a plot of ET rate constant versus substrate potential using analytical equations developed in this work. A pair of such plots thus obtained from a pair of \(i_T-E_S\) voltammograms in feedback and SG/TC modes reveal the
potential dependence of ET rate constants for both anodic and cathodic processes at the substrate. This analytical approach facilitates applications of $i_T-E_S$ voltammetry to non-BV systems based on electrocatalysis, $^{53}$ charge-transfer at liquid/liquid interfaces, $^{54}$ nanoparticle-mediated ET at gold electrodes modified with self-assembled monolayers, $^{55}$ and ET at single-walled carbon nanotubes and graphenes. $^{28}$ In fact, nanoscale mass transport is inevitable when individual single-walled carbon nanotubes are investigated as SECM substrates. $^{20}$

**Supporting Information.** Additional information as noted in text. This material is available free of charge via the Internet at http://pubs.acs.org.

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